**Polynomials**

1. A **polynomial** *p(x)* in one variable *x* is an algebraic expression in *x* of the form

*p(x)* = *a xn*  *a xn*1  *a xn*2  ........  *a x*2  *a x*  *a* , where

*n n*1

* 1. *a*0 ,*a*1,*a*2 *an*

*n*2 2 1 0

are constants

* 1. x is a variable
  2. a0 , a1, a2......an are respectively the **coefficients** of xi
  3. Each of *a xn* ,*a xn*1,*a xn*2,........*a x*2,*a x*,*a* , with *a*  0, is called a **term** of a polynomial.

*n n*1 *n*2 2 1 0 *n*

1. The highest power of the variable in a polynomial is called the **degree** of the polynomial.
2. A polynomial with one term is called a **monomial**.
3. A polynomial with two terms is called a **binomial**.
4. A polynomial with three terms is called a **trinomial**.
5. A polynomial with degree zero is called a **constant polynomial**. For example: 1, -3. The degree of non-zero constant polynomial is zero
6. A polynomial of degree one is called a **linear polynomial**. It is of the form *ax* + *b*. For example: *x* - 2, 4*y* + 89, 3*x - z*.
7. A polynomial of degree two is called a **quadratic polynomial**. It is of the form *ax2 + bx + c*. where *a, b,*

*c* are real numbers and *a* 0 For example: *x*2  2*x*  5 etc.

1. A polynomial of degree three is called a **cubic polynomial** and has the general form *ax3 + bx2* + *cx +d*.

For example:

*x*3  2*x*2  2*x*  5

etc.

1. A **bi-quadratic polynomial** *p(x*) is a polynomial of degree four which can be reduced to quadratic polynomial in the variable *z = x*2 by substitution.
2. The constant polynomial 0 is called the **zero polynomial**. Degree of zero polynomial is not defined.
3. The **value of a polynomial** f(x) at *x* = *p* is obtained by substituting *x* = *p* in the given polynomial and is denoted by *f*(*p*).
4. A real number ‘*a*’ is a **zero** or root of a polynomial *p*(*x*) if *p* (*a*) = 0.
5. The number of real zeroes of *a* polynomial is less than or equal to the degree of polynomial.
6. Finding a zero or root of a polynomial f(x) means solving the polynomial equation *f*(*x*) = 0.
7. A non-zero constant polynomial has no zero.
8. Every real number is a zero of a zero polynomial.

# Division algorithm

If *p(x*) and *g(x*) are the two polynomials such that degree of *p(x)*  degree of *g(x)* and g*(x) ≠* 0, then we can find polynomials *q(x*) and *r(x)* such that:

*p (x)* = *g(x) q(x) + r(x)*

where, *r(x)* =0 or degree of *r(x)* < degree of *g(x)*.

# Remainder theorem

Let *p*(*x*) be any polynomial of degree greater than or equal to one and let a be any real number. If *p*(*x*) is divided by the linear polynomial (*x – a*), then remainder is *p*(*a*).

* 1. If the polynomial *p(x*) is divided by (*x + a*), the remainder is given by the value of *p* (-*a*).
  2. If p(x) is divided by *ax + b =* 0*; a*  *0*, the remainder is given by

*p*  *b*  ; *a*  0

 *a* 

 

* 1. If *p (x)* is divided by *ax - b = 0* , *a*  0 , the remainder is given by

*p*  *b*  ; a  0

 *a* 

 

# Factor theorem

Let *p*(*x*) is a polynomial of degree *n* ≥ 1 and *a* is any real number such that *p*(*a*) = 0, then (*x* - *a*) is a factor of *p*(*x*).

# Converse of factor theorem

Let *p*(*x*) is a polynomial of degree *n* ≥ 1 and *a* is any real number. If (*x* - *a*) is a factor of *p*(*x*), then

*p*(*a*) = 0.

* 1. (*x* + *a*) is a factor of a polynomial *p*(*x*) iff *p*(-*a*) = 0.
  2. (*ax* - *b*) is a factor of a polynomial *p*(*x*) iff *p*(*b*/*a*) = 0.
  3. (*ax* + *b*) is a factor of a polynomial *p*(*x*) iff *p*(-*b*/*a*) = 0.
  4. (*x* - *a*)(*x* - *b*) is a factor of a polynomial *p*(*x*) iff *p*(*a*) = 0 and *p*(*b*) = 0.

1. For applying factor theorem, the divisor should be either a linear polynomial of the form (*ax* + *b*) or it should be reducible to a linear polynomial.
2. A quadratic polynomial *ax2 + bx+ c* is **factorised by splitting the middle term** by writing *b* as *ps + qr*

such that (*ps)* (*qr*) *= ac*.

Then, *ax2 + bx+ c* = (px + q) (rx + s)

1. An **algebraic identity** is an algebraic equation which is true for all values of the variables occurring in it.
2. Some useful **quadratic identities**: i. *x*  *y* 2  *x*2  2*xy*  *y* 2
3. *x*  *y* 2  *x*2  2*xy*  *y* 2
4. *x*  *y* (*x*  *y* )  *x*2  *y* 2
5. *x*  *a*(*x*  *b*)  *x*2  (*a*  *b*)*x*  *ab*

v. *x*  *y*  *z*2  *x*2  *y* 2  *z*2  2*xy*  2*yz*  2*zx*

Here *x, y, z* are variables and *a, b* are constants.

1. Some useful **cubic identities**:

i. *x*  *y* 3  *x*3  *y* 3  3*xy*(*x*  *y* )

ii. *x*  *y* 3  *x*3  *y* 3  3*xy*(*x*  *y* )

iii. *x*3  *y* 3  (*x*  *y* )(*x*2  *xy*  *y* 2 )

iv. *x*3  *y* 3  (*x*  *y* )(*x*2  *xy*  *y* 2 )

v. *x*3  *y*3  *z*3  3*xyz*  (*x*  *y*  *z*)(*x*2  *y* 2  *z*2  *xy*  *yz*  *zx*)

vi. If

*x*  *y*  *z*  0 then

*x*3  *y* 3  *z*3  3*xyz*

Here, *x, y* and *z* are variables.